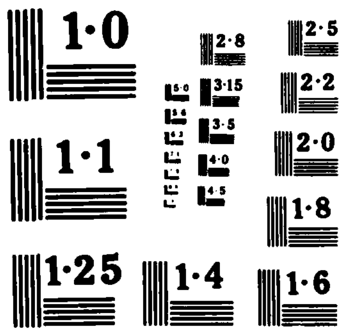
AD-A144 860 ON THE DISTRIBUTION OF CYCLES TO CRACK INITIATION(U) 1/1
GEORGE WASHINGTON UNIV WASHINGTON DC INST FOR
RELIABILITY AND RISK ANALYSIS R SOYER 08 APR 83
UNCLASSIFIED GWU/IRRA/TR-83/1 N00019-82-C-0458 F/G 20/11 NL

END
FILMED
DTIC


1·0
2·8
2·5
3·15
2·2
3·5
2·0
1·1
4·0
4·5
1·8
1·25
1·4
1·6

(12)

AD-A144 860

ON THE DISTRIBUTION OF CYCLES TO CRACK INITIATION

by

Refik Soyer

GWU/IRRA/TR-83/1
8 April 1983

The George Washington University
School of Engineering and Applied Science
Institute for Reliability and Risk Analysis

Research Supported by
NAVAIR
Contract N00019-82-C-0458

ON THE DISTRIBUTION OF CYCLES TO CRACK INITIATION

by

Refik Soyer

GWU/IRRA/TR-83/1
8 April 1983

The George Washington University
School of Engineering and Applied Science
Institute for Reliability and Risk Analysis

Research Supported by

NAVAIR
Contract N00019-82-C-0458

THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Institute for Reliability and Risk Analysis

Abstract
of
GWU/IRRA/TR-83/1
8 April 1983

ON THE DISTRIBUTION OF CYCLES TO CRACK INITIATION

by

Refik Soyer

This report is a supplement to our previous report on the distribution of the number of cycles to crack initiation in aircraft engine disks. The data analyzed in the previous report are reconsidered here using a new approach. Thus, some degree of familiarity with the first report is assumed. We first present a nonparametric approach for estimating the survival (reliability) function. This technique is applied to the actual data, and comparisons are made with the survival curves that are obtained assuming the lognormal, the Weibull, and the inverse Gaussian distributions. An alternative informal goodness of fit test for the inverse Gaussian distribution is also considered. Our conclusion is that estimated survival curves using the lognormal, the Weibull, and the inverse Gaussian distributions behave similarly, whereas the survival curve based on a nonparametric approach is more conservative as compared to the other curves.

Research Supported by
NAVAIR

ACKNOWLEDGMENTS

The work that is reported here was performed under the advice of Professor Nozer D. Singpurwalla. His helpful comments and advice for the preparation of the report are acknowledged. Also acknowledged are the comments of Mr. Jerzy Kyparisis in connection with the writing of this report.

THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Institute for Reliability and Risk Analysis

ON THE DISTRIBUTION OF CYCLES TO CRACK INITIATION

by

Refik Soyer

1. OVERVIEW OF THE APPROACH USED

In our first report [Soyer (1982)], the problem of choosing a distribution for the number of cycles to crack initiation was analyzed using plotting techniques and some formal goodness of fit tests. The techniques for the Weibull, the lognormal, and the inverse Gaussian distributions that were known to us at that time were discussed in detail.

In this report, the same problem, namely, the choice of distribution, is considered, but by using a different approach. A new method for estimating the survival function, suggested by Proschan (1982), is applied to the same data. The estimated survival function called "non-parametric" survival function, is compared with the survival functions obtained assuming different parametric life models.

In Section 3 an alternate informal goodness of fit test for the inverse Gaussian distribution is discussed and applied to the data.

2. THE NONPARAMETRIC APPROACH

In our first report, three life models, the Weibull, the lognormal, and the inverse Gaussian, were analyzed.

The survival functions for these models are of the following forms.

The *Weibull model*

$$\bar{F}(x) = \exp\left\{-\left(\frac{x}{\delta}\right)^{\beta}\right\}, \quad x \geq 0 \text{ , } \beta,\delta > 0 \text{ .} \tag{2.1}$$

The *lognormal model*

$$\bar{F}(x) = 1 - \Phi\left(\frac{\ell nx-\mu}{\sigma}\right), \quad x \geq 0 \text{ , } \mu,\sigma > 0 \text{ ,} \tag{2.2}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal.

The *inverse Gaussian model*

$$\bar{F}(x) = 1 - \left\{\Phi\left(\sqrt{\lambda/x}\ [(x/\mu)-1]\right) + e^{2\lambda/\mu}\ \Phi\left(-\sqrt{\lambda/x}\ [1+(x/\mu)]\right)\right\}$$

or

$$\bar{F}(x) = \Phi\left(\sqrt{\lambda/x}\ [1-(x/\mu)]\right) - e^{2\lambda/\mu}\ \Phi\left(-\sqrt{\lambda/x}\ [1+(x/\mu)]\right) \text{ .} \tag{2.3}$$

To estimate the survival function, the parameters of the assumed distribution are estimated from the data, and the estimated values plugged in to the above equations.

The maximum likelihood estimates of the parameters of the models are given below.

<u>The Weibull Model</u>. Estimation of parameters β and δ using the maximum likelihood method requires the solution of the following equations:

$$0 = \sum_{i=1}^{N} [-\beta/\delta + (\beta/\delta)(x_i/\delta)^{\beta}] \tag{2.4}$$

$$0 = \sum_{i=1}^{N} [(1/\beta) + \ell n(x_i/\delta) - (x_i/\delta)^{\beta}\ \ell n(x_i/\delta)] \text{ .} \tag{2.5}$$

These equations are solved for δ and β by using an iterative procedure and the estimates $\hat{\delta}$ and $\hat{\beta}$ are obtained.

The Lognormal Model. The maximum likelihood estimates of μ and σ^2 are:

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} Y_i$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{N} (Y_i)^2 - N\left[\sum_{i=1}^{N} Y_i/N\right]^2}{N}, \qquad (2.6)$$

where $Y_i = \ln X_i$.

The Inverse Gaussian Model. The maximum likelihood estimates are:

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} X_i = \bar{X}$$

$$\hat{\lambda}^{-1} = \frac{1}{N} \sum_{i=1}^{N} \left[\frac{1}{X_i} - \frac{1}{\bar{X}}\right] \qquad (2.7)$$

2.1 Nonparametric Estimation

In a recent paper, Proschan presents a method for estimating the survival function. The method is applicable to both censored and uncensored life data.

The method first estimates the average "failure rate" on each interval between successive failures. Then a survival function is obtained for each interval, and the separate pieces are joined to form a continuous function. The continuous function gives the estimated values of the survival function, including the point at which last failure is observed. It does not provide any estimate of the survival function beyond the last observed failure.

The failure rate, $\pi(x)$, is defined

$$\pi(x) = \frac{f(x)}{1 - F(x)} = \frac{f(x)}{\bar{F}(x)} ,$$

where $F(x)$ is the distribution function of the random variable X, the time to failure (or cycles to failure), and $f(x)$ is its probability density function.

The failure rate has a probabilistic interpretation, namely, $\pi(x)dx$ represents the probability that a device of age x will fail in the interval $(x, x+dx)$ [see Mann, Schafer, and Singpurwalla (1974)].

In Proschan (1982), an estimate of the failure rate in the interval $(z_{i-1}, z_i]$, where z_i and z_{i-1} denote the time of the ith and (i-1)st observed failures, is given as

$$\hat{\pi}_i = \frac{1}{\text{observed total time on test}} . \qquad (2.8)$$

For our purposes, the definition of total time on test will be given for the case of uncensored samples. In Proschan, it is defined for the general case, where the withdrawals are taken into account.

For uncensored data, the observed total time on test is defined as:

$$\begin{aligned} TTOT_i &= (z_i - z_{i-1}) + (N-i)(z_i - z_{i-1}) \\ &= (N-i+1)(z_i - z_{i-1}) , \quad z_0 = 0 , \end{aligned} \qquad (2.9)$$

where N denotes the number of items on test.

By using (2.8) the failure rates on successive intervals can be estimated. Once all the estimates are obtained, we can use $\hat{\bar{F}}$, the estimator of the survival function given by Proschan (1982) as

$$\hat{\bar{F}} = \begin{cases} \exp\{-\hat{\pi}_1 x\}, & 0 \leq x \leq z_1 \\ \exp\{-[\hat{\pi}_1 z_1 + \hat{\pi}_2(x-z_1)]\}, & z_1 < x \leq z_2 \\ \exp\{-[\hat{\pi}_1 z_1 + \hat{\pi}_2(z_2-z_1) + \ldots + \hat{\pi}_i(z_i - z_{i-1}) + \hat{\pi}_{i+1}(x-z_i]\}, & \\ & z_i < x \leq z_{i+1} \; ; \; i = 1,2,\ldots,N-1 . \end{cases} \tag{2.10}$$

As stated before, there is no estimator for $x > z_N$.

2.2 Application of the Method

In this section the results of Section 2.1 are applied to the data presented in Tables A.1 and A.2 of the Appendix and the nonparametric survival function is estimated. The nonparametric survival function is then compared with the survival functions estimated assuming different life models.

2.2.1 Analysis of the first set of data

The nonparametric estimation method is applied to the data presented in Table A.1. The application of Equation (2.8) to successive intervals $(0,z_1]$, $(z_1,z_2]$, ..., $(z_5,z_6]$ gives us the estimates of the failure rate on each interval. The estimated failure rates, $\hat{\pi}_i$'s , $i = 1,2,\ldots,6$, are presented in Table 1. The estimate of the survival function, $\hat{\bar{F}}$, is obtained using Equation (2.10). The estimated survival function, $\hat{\bar{F}}$, is presented in Figure 1.

A comparison of the nonparametric survival function with the survival functions obtained from different life models is a way of checking the appropriateness of the life model assumed.

Table 1: Estimated Failure Rates

Flight Hours to Failure z_i	Estimated Failure Rate $\hat{\pi}_i$	Failure Rate Interval $(z_{i-1}, z_i]$
780	0.00021	(0, 780]
820	0.00500	(780, 820]
910	0.00278	(820, 910]
950	0.00833	(910, 950]
1050	0.02000	(950, 1050]

The estimation of the survival functions assuming different life models has already been discussed. The maximum likelihood estimates of the parameters of the models and the estimated survival functions are presented in Tables 2 and 3, respectively.

The comparison of the columns of Table 3 shows that the lognormal (column 3) and the inverse Gaussian (column 4) survival functions behave similarly, whereas the Weibull survival function (column 2) differs slightly from the other two. A comparison of these three models with the nonparametric survival function is made in Figures 2 and 3. An inspection of the figures indicates that the three models seem equally

Table 2: Maximum Likelihood Estimates of the Parameters of the Weibull, the Lognormal, and the Inverse Gaussian Distributions

Weibull	Lognormal	Inverse Gaussian
$\hat{\beta}$ = 10.54	$\hat{\mu}$ = 6.83	$\hat{\mu}$ = 926.67
$\hat{\delta}$ = 972.93	$\hat{\sigma}^2$ = 0.013	$\hat{\lambda}$ = 72256.10

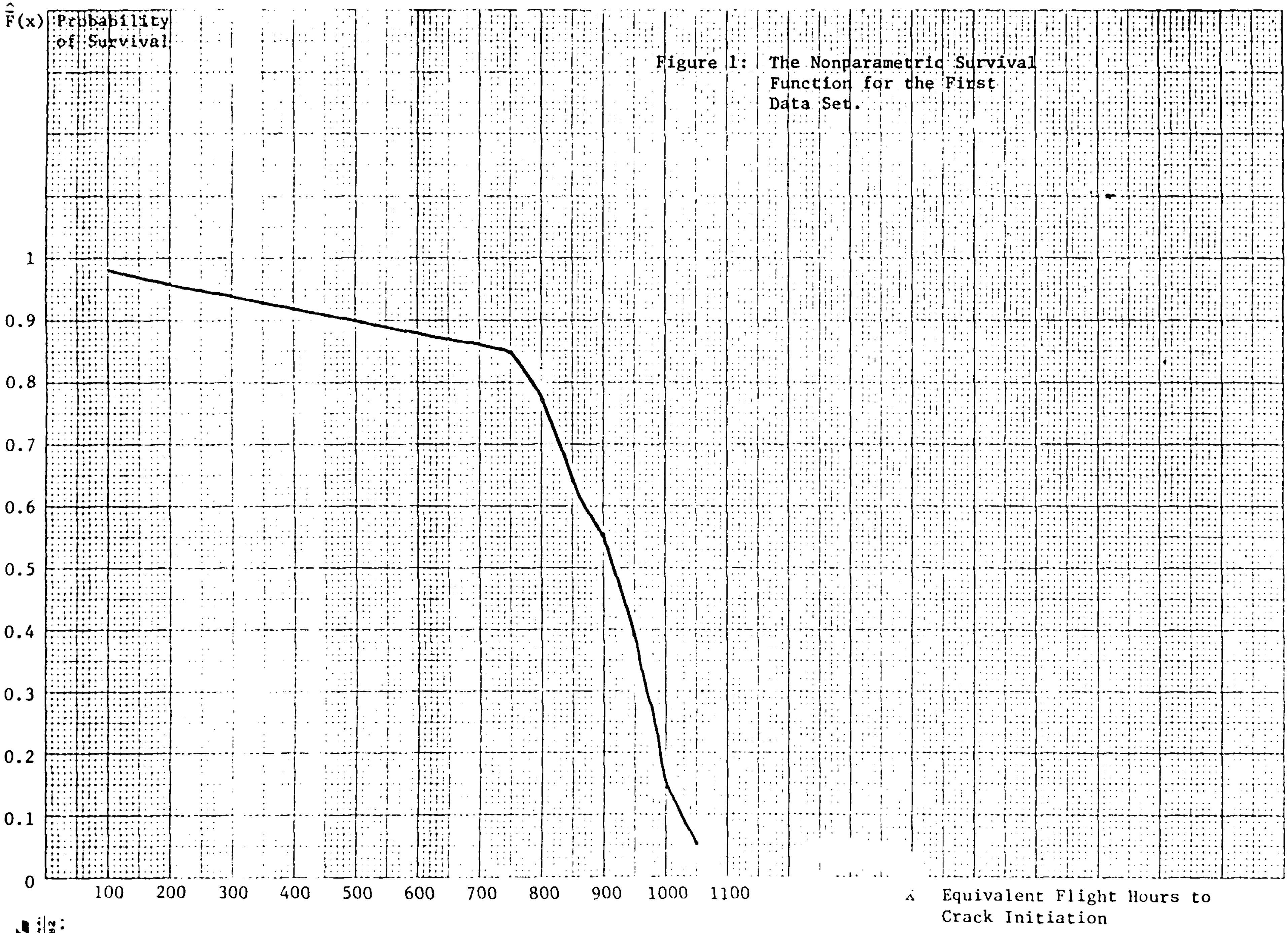


Figure 1: The Nonparametric Survival Function for the First Data Set.

Table 3: Survival Functions for the Weibull, the Lognormal, and the Inverse Gaussian Distribution

(1) X	(2) $\bar{F}(X)_W$	(3) $\bar{F}(X)_{LN}$	(4) $\bar{F}(X)_{IG}$
500	1.00	1.00	1.00
550	0.99	1.00	1.00
600	0.99	0.99	0.99
650	0.98	0.99	0.99
700	0.97	0.99	0.99
750	0.94	0.96	0.97
800	0.88	0.90	0.90
850	0.79	0.77	0.78
900	0.64	0.59	0.60
950	0.46	0.41	0.41
1000	0.26	0.24	0.25
1050	0.11	0.13	0.14

appropriate for describing the data and that the nonparametric model is more conservative than the others.

2.2.2 Analysis of the second set of data

The data which will be analyzed in this section are presented in Table A.2 of the Appendix. The nonparametric estimation procedure is applied to the data. The estimated failure rates, the $\hat{\pi}_i$'s , are presented in Table 4.

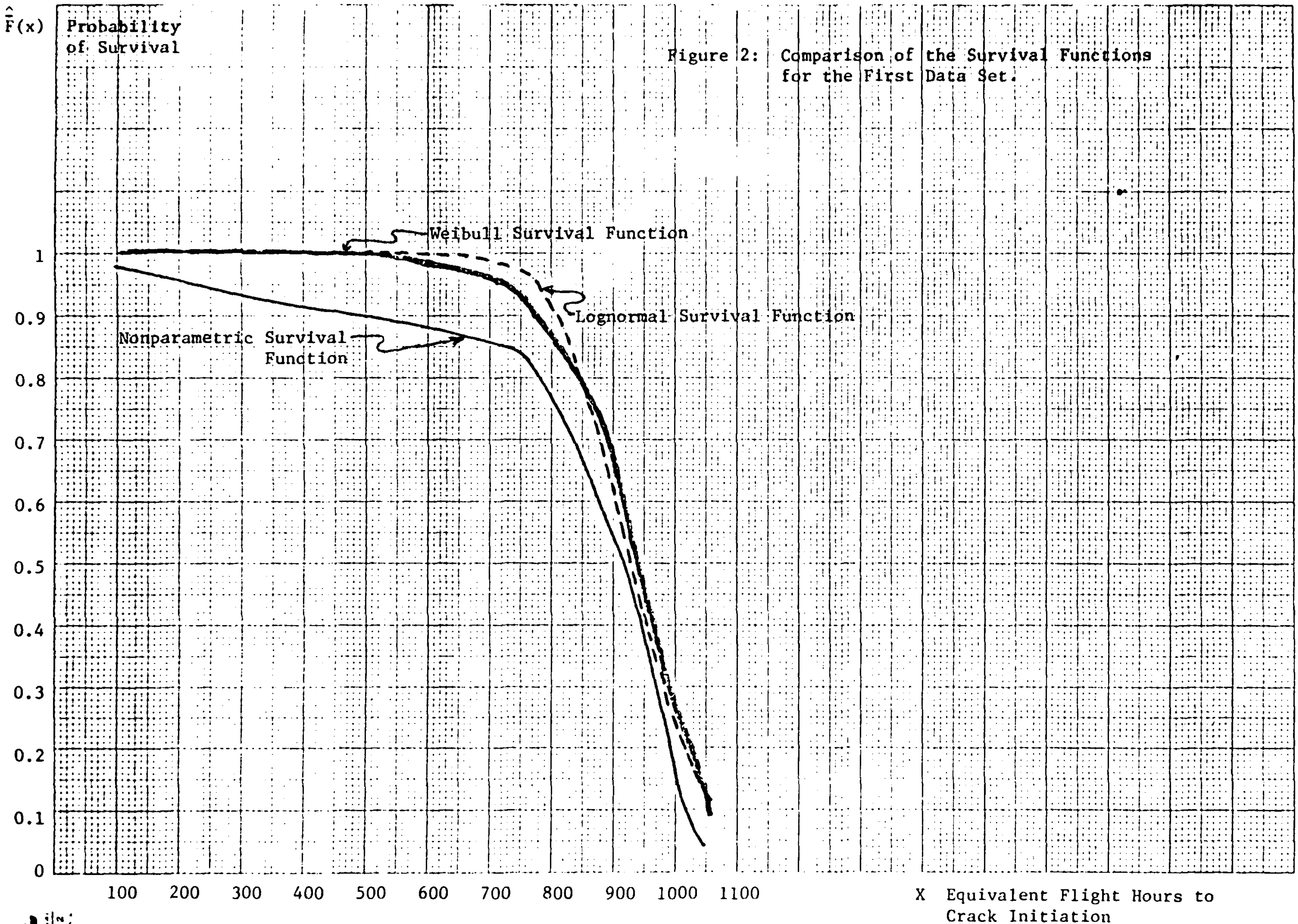


Figure 2: Comparison of the Survival Functions for the First Data Set.

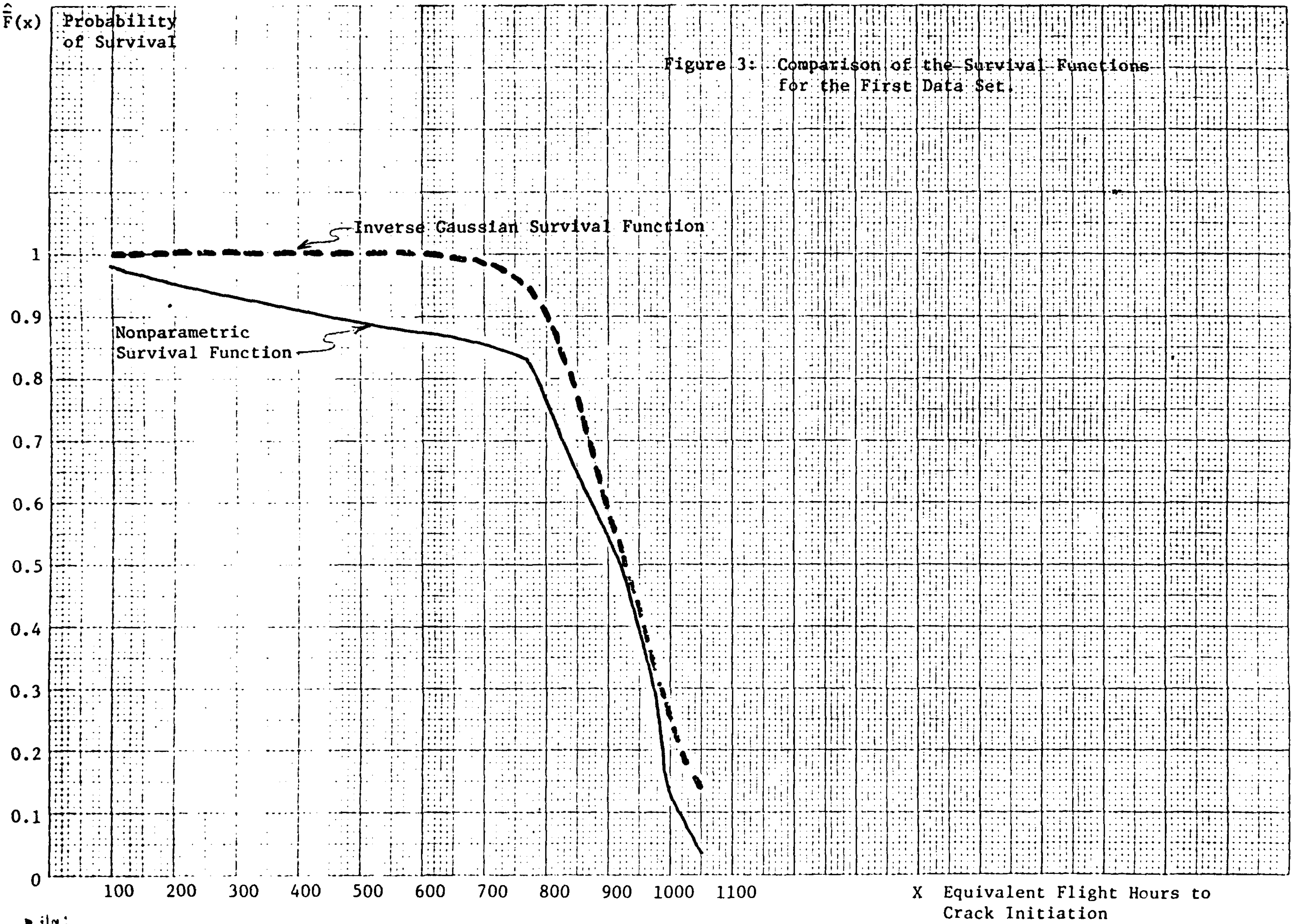


Figure 3: Comparison of the Survival Functions for the First Data Set.

Table 4: Estimated Failure Rates

Cycles to Failure z_i	Estimated Failure Rate $\hat{\pi}_i$	Failure Rate Interval $(z_{i-1}, z_i]$
1066	0.00005	(0, 1066]
1078	0.00556	(1066, 1078]
1100	0.00324	(1078, 1100]
1188	0.00087	(1100, 1188]
1214	0.00321	(1188, 1214]
1240	0.00349	(1214, 1240]
1310	0.00143	(1240, 1310]
1416	0.00105	(1310, 1416]
1459	0.00291	(1416, 1459]
1624	0.00087	(1459, 1624]
1660	0.00463	(1624, 1660]
1670	0.02000	(1660, 1670]
1679	0.02778	(1670, 1679]
1706	0.01235	(1679, 1706]
1738	0.01563	(1706, 1738]
1780	0.02381	(1738, 1780]

The nonparametric survival function for these data is presented in Figure 4.

The maximum likelihood estimates of the parameters of the three life models and their estimated survival functions are presented in Tables 5 and 6, respectively.

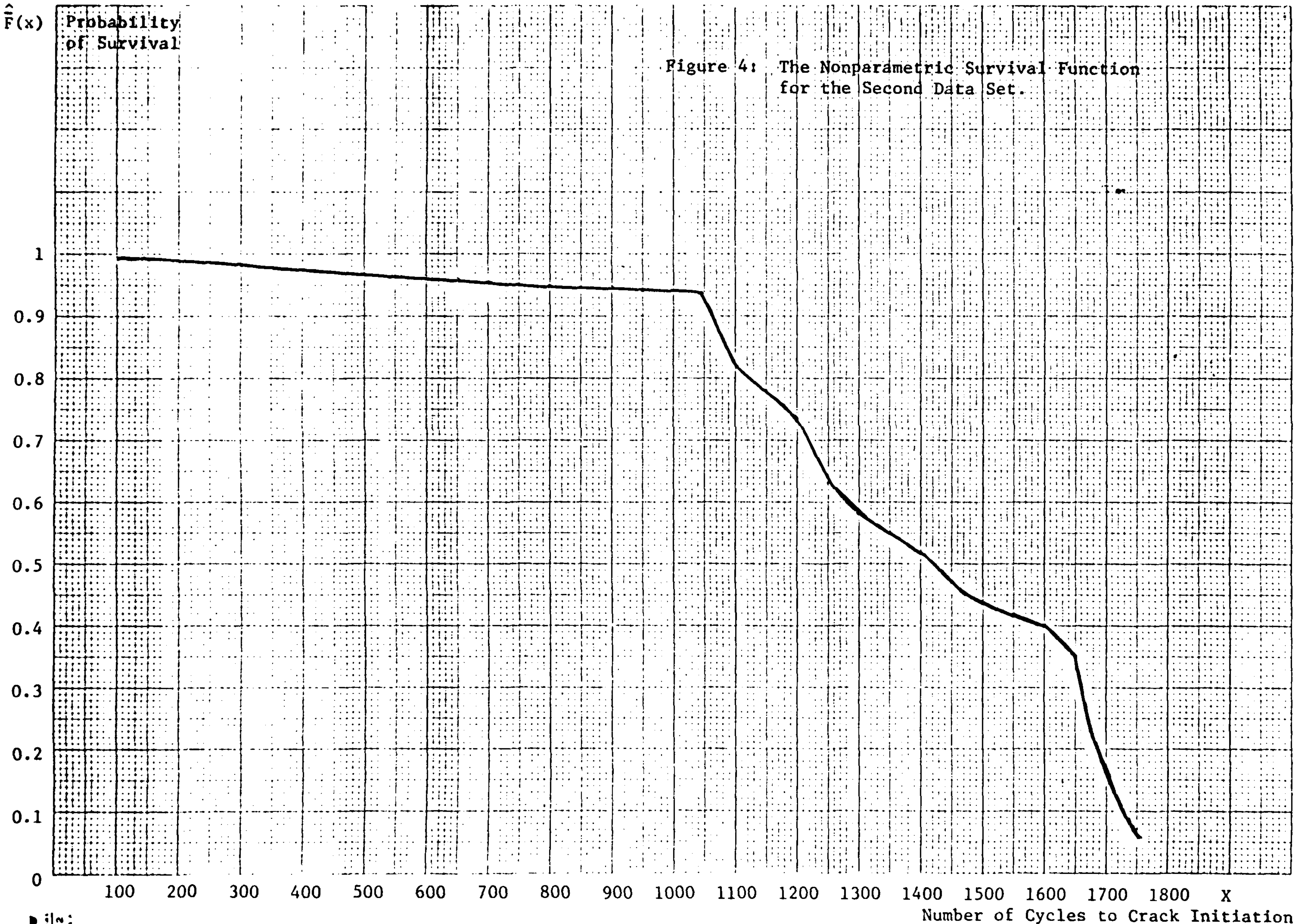


Figure 4: The Nonparametric Survival Function for the Second Data Set.

Table 5: Maximum Likelihood Estimates of the Parameters of the Weibull, the Lognormal, and the Inverse Gaussian Distributions

Weibull	Lognormal	Inverse Gaussian
$\hat{\beta}$ = 6.71	$\hat{\mu}$ = 7.25	$\hat{\mu}$ = 1433
$\hat{\delta}$ = 1539.60	$\hat{\sigma}^2$ = 0.033	$\hat{\lambda}$ = 42983

Table 6: Survival Functions for the Weibull, the Lognormal, and the Inverse Gaussian Distributions

(1) X	(2) $\bar{F}(x)_W$	(3) $\bar{F}(x)_{LN}$	(4) $\bar{F}(x)_{IG}$
500	1.00	1.00	1.00
600	0.99	1.00	1.00
700	0.99	1.00	1.00
800	0.98	1.00	1.00
900	0.97	0.99	0.99
1000	0.95	0.97	0.97
1100	0.90	0.91	0.92
1200	0.83	0.81	0.83
1300	0.72	0.67	0.70
1400	0.59	0.51	0.54
1500	0.43	0.36	0.40
1600	0.27	0.24	0.26
1700	0.14	0.15	0.17
1750	0.09	0.11	0.13

The comparison of the columns of Table 6 shows results similar to those obtained in Section 2.2.1. A comparison of the life models with the nonparametric survival function is made in Figures 5 and 6. These two figures show that the three life models behave similarly and that the nonparametric model is initially more conservative than the others.

3. ANOTHER INFORMAL GOODNESS-OF-FIT TEST FOR THE INVERSE GAUSSIAN MODEL

In our first report, a distribution-free Kolmogorov-Smirnov test, assuming the parameters were known, was used to test for goodness of fit for the inverse Gaussian distribution. It was stated there that a formal goodness of fit test for this distribution was not known. However, some properties of the inverse Gaussian distribution and its relation with the normal distribution suggest the possibility of using the formal goodness-of-fit tests for the normal distribution, if certain conditions are satisfied.

3.1 Derivation of Inverse Gaussian Distribution

The derivation of inverse Gaussian distribution is discussed in detail in Chhikara and Folks (1974).

If a random variable X has an inverse Gaussian distribution, then its density function is

$$f(x;\mu,\lambda) = \left(\frac{\lambda}{2\pi x^3}\right)^{1/2} \exp\left\{-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right\}, \quad x > 0 . \tag{3.1}$$

If Y is another random variable, such that

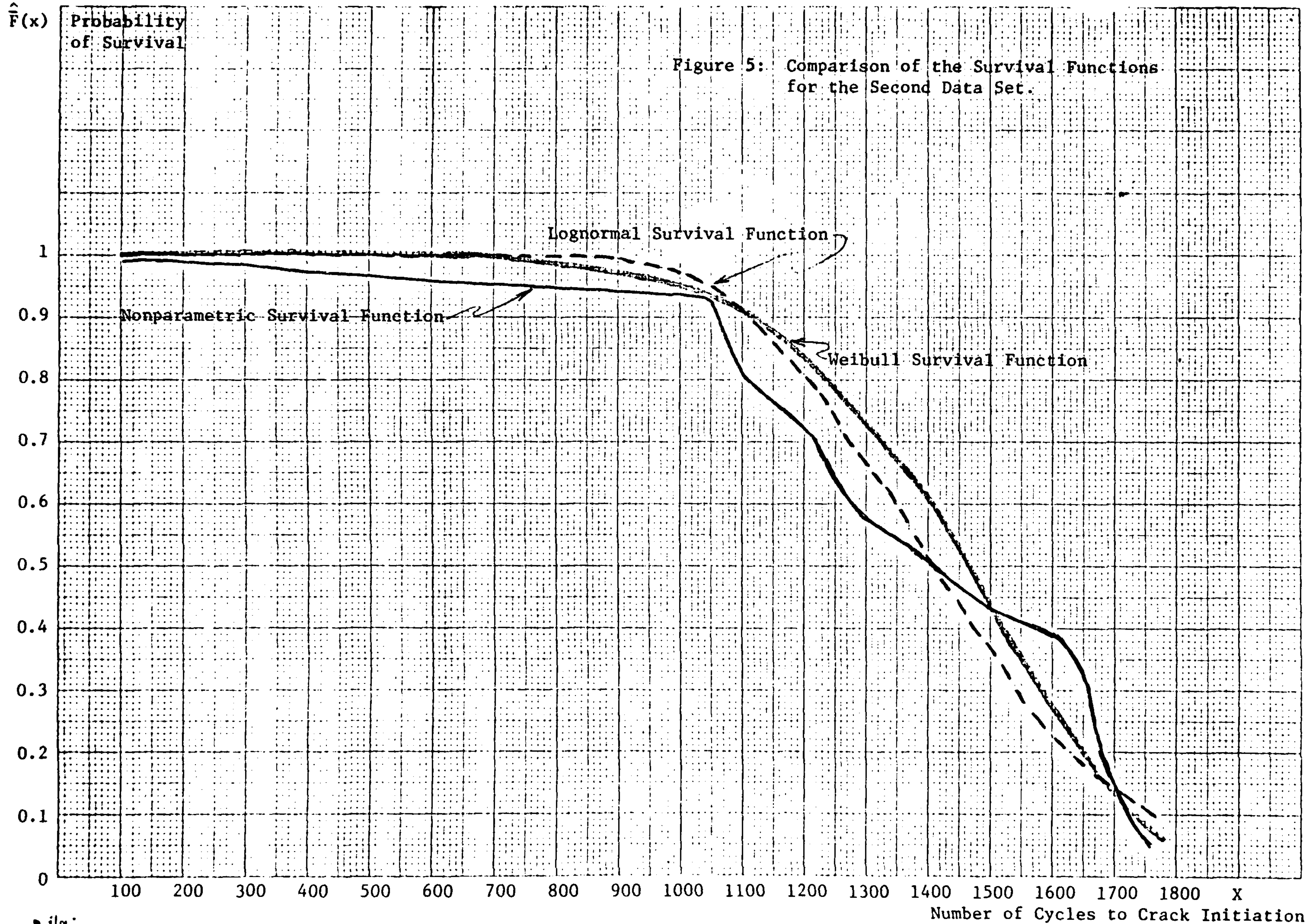


Figure 5: Comparison of the Survival Functions for the Second Data Set.

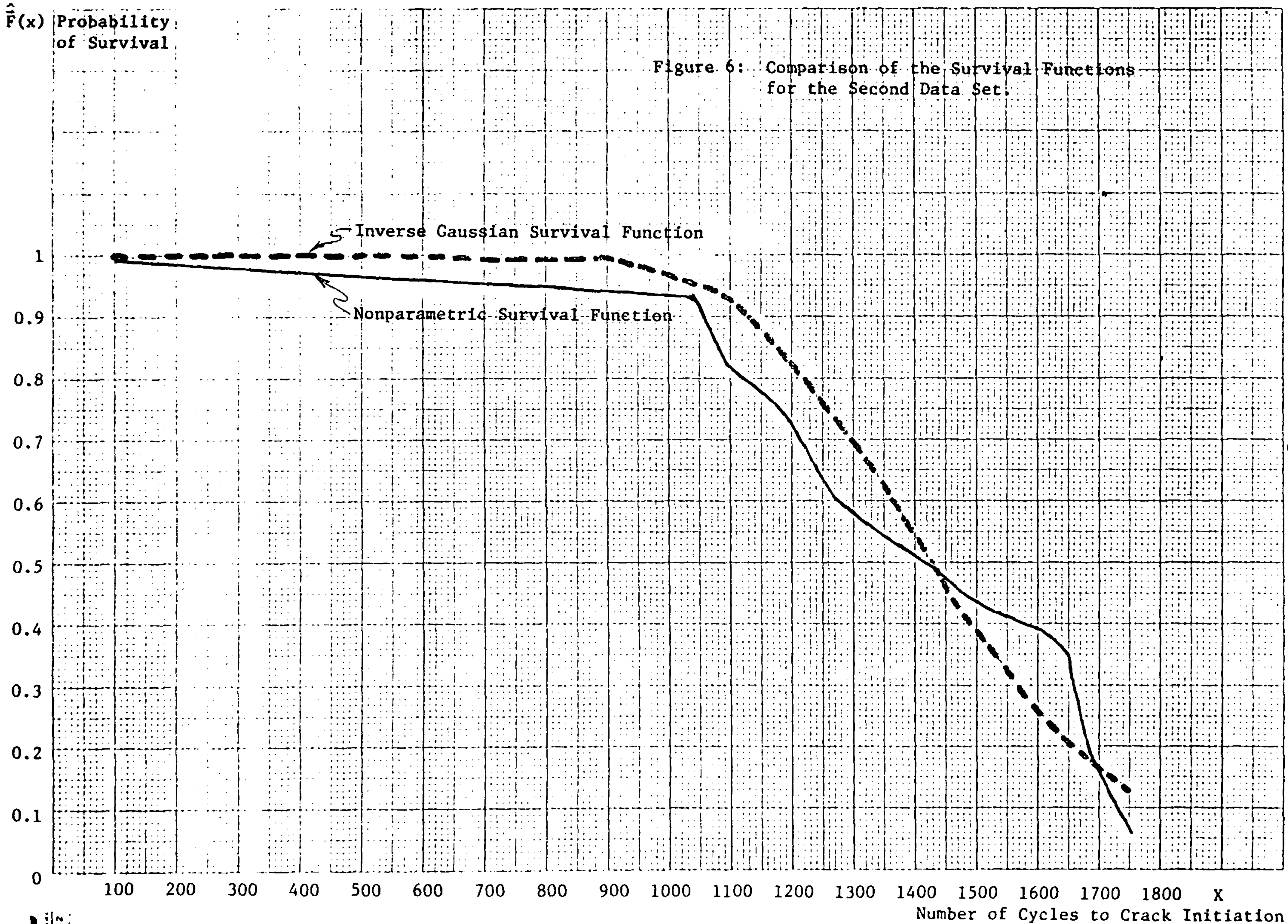


Figure 6: Comparison of the Survival Functions for the Second Data Set.

$$Y = \frac{\sqrt{\lambda}\ (x-\mu)}{\mu\sqrt{x}} , \tag{3.2}$$

then the density of Y can be obtained as:

$$g(y;\lambda/\mu) = \left(1 - \frac{y}{\sqrt{4\lambda/\mu+y^2}} \right)\left(\frac{1}{\sqrt{2\pi}} e^{-y^2/2} \right) \tag{3.3}$$

where $-\infty < y < \infty$.

As Chhikara and Folks (1975) state, the distribution function of Y is:

$$F(y) = \Phi(y) + e^{2\lambda/\mu}\, \Phi\left(- \sqrt{4\lambda/\mu + y^2} \right) , \tag{3.4}$$

where $\Phi(\cdot)$ denotes the standard normal distribution function. We note that if $\lambda/\mu \to \infty$, then (3.4) becomes the standard normal distribution function. The transformation (3.2) is one-to-one; therefore, the survival function of X can be obtained as in (2.3).

Thus, if the ratio λ/μ gets large, we can use some goodness of fit tests that are available for the normal distribution.

3.2 Application to the Data

For the first set of data which was analyzed in Section 2.2.1, the maximum likelihood estimates of λ and μ were presented in Table 2. For this data set the ratio $\hat{\lambda}/\hat{\mu}$ is approximately equal to 78. For this value, the term $\Phi\left(- \sqrt{4\lambda/\mu + y^2}\right) \to 0$ in (3.4) and the distribution function of Y becomes $\Phi(y)$. Thus, we can use one of the goodness of fit tests for the normal distribution.

The Anderson-Darling goodness-of-fit test has the statistic

$$A^2 = -\left\{ \sum_{i=1}^{n} (2i-1)\ [\ell n\ F(x_i) + \ell n[1 - F(x_{n+1-i})]] \right\} / n - n .$$

For the first set of data the test statistic is computed as $A^2 = 0.33$ which requires the acceptance of the null hypothesis that the data are from the inverse Gaussian distribution.

For the second set of data which was analyzed in Section 2.2.2, the ratio $\hat{\lambda}/\hat{\mu}$ is approximately equal to 30. For this value, $\phi(-\sqrt{4\lambda/\mu+y^2}) \to 0$ in (3.4) and the distribution function of Y becomes standard normal. Thus, we can use the Anderson-Darling test for the second set of data. The test statistic is computed as $A^2 = 0.76$, which requires the acceptance of the null hypothesis that the sample is from the inverse Gaussian distribution.

4. CONCLUSION

The application of the nonparametric estimation approach to the two data sets indicates that the nonparametric model behaves more conservatively than the parametric life models considered in the analysis. The Weibull, lognormal, and inverse Gaussian models behave similarly and therefore they are equally appropriate as life models.

The results obtained in Section 3 indicate that the formal goodness of fit tests for the normal distribution can be used for the inverse Gaussian model if certain conditions are satisfied.

On the basis of the present analysis, it can be concluded that the nonparametric model which behaves more conservatively than the others may be considered as an alternative model for describing the distribution of the number of cycles to crack initiation.

REFERENCES

Chhikara, R. S. and J. L. Folks (1974). Estimation of the inverse Gaussian distribution function. *Journal of the American Statistical Association*, Vol. 69, pp. 250-254.

Chhikara, R. S. and J. L. Folks (1975). Statistical distributions related to the inverse Gaussian. *Communications in Statistics*, Vol. 4, No. 12, pp. 1081-1091.

Chhikara, R. S. and J. L. Folks (1977). The inverse Gaussian distribution as a lifetime model. *Technometrics*, Vol. 19, pp. 461-468.

Dyer, A. R. (1974). Comparisons of tests for normality with a cautionary note. *Biometrika*, Vol. 61, pp. 185-189.

Mann, N. R., R. E. Schafer, and N. D. Singpurwalla (1974). *Methods for Statistical Analysis of Reliability and Life Data*. Wiley, New York.

Proschan, F. (1982). Piecewise exponential estimator. Report in progress.

Soyer, R. (1982). Distribution of the number of cycles to fatigue crack initiation in aircraft engine disks. Technical Report GWU/IRRA/TR-82/6, Institute for Reliability and Risk Analysis, The George Washington University.

Wasan, M. T. and L. K. Roy (1969). Tables of inverse Gaussian percentage points. *Technometrics*, Vol. 11, pp. 591-604.

APPENDIX

TABLES

TABLE A.1

CRACK INITIATION IN 10TH COMPRESSOR DISK

Disk Number	Sawtooth Cycles	Equivalent Flight Hours
1	1275	1050
2	1150	950
3	1000	820
4	1270	1050
5	950	780
6	1100	910

TABLE A.2

CYCLES TO CRACK INITIATION
IN BOLT-HOLES

Bolt-hole Number	Sawtooth Cycles
1	1214
2	1188
3	1240
4	1078
5	1066
6	1100
7	1670
8	1660
9	1679
10	1310
11	1459
12	1624
13	1738
14	1706
15	1416
16	1780

END

FILMED

10-84

DTIC